



7th Iranian Geometry Olympiad
Intermediate level
October 30, 2020

The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.ir

Problem 1. A trapezoid $ABCD$ is given where AB and CD are parallel. Let M be the midpoint of the segment AB . Point N is located on the segment CD such that $\angle ADN = \frac{1}{2}\angle MNC$ and $\angle BCN = \frac{1}{2}\angle MND$. Prove that N is the midpoint of the segment CD .

Problem 2. Let ABC be an isosceles triangle ($AB = AC$) with its circumcenter O . Point N is the midpoint of the segment BC and point M is the reflection of the point N with respect to the side AC . Suppose that T is a point so that $ANBT$ is a rectangle. Prove that $\angle OMT = \frac{1}{2}\angle BAC$.

Problem 3. In acute-angled triangle ABC ($AC > AB$), point H is the orthocenter and point M is the midpoint of the segment BC . The median AM intersects the circumcircle of triangle ABC at X . The line CH intersects the perpendicular bisector of BC at E and the circumcircle of the triangle ABC again at F . Point J lies on circle ω , passing through X , E , and F , such that $BCHJ$ is a trapezoid ($CB \parallel HJ$). Prove that JB and EM meet on ω .

Problem 4. Triangle ABC is given. An arbitrary circle with center J , passing through B and C , intersects the sides AC and AB at E and F , respectively. Let X be a point such that triangle FXB is similar to triangle EJC (with the same order) and the points X and C lie on the same side of the line AB . Similarly, let Y be a point such that triangle EYC is similar to triangle FJB (with the same order) and the points Y and B lie on the same side of the line AC . Prove that the line XY passes through the orthocenter of the triangle ABC .

Problem 5. Find all numbers $n \geq 4$ such that there exists a convex polyhedron with exactly n faces, whose all faces are right-angled triangles.
(Note that the angle between any pair of adjacent faces in a convex polyhedron is less than 180° .)

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.